

Noncommutative Geometry and Supergravity.

José Luis López,^{*} O. Obregón,[†] and M. Sabido[‡]

*Departamento de Física de la Universidad de Guanajuato,
A.P. E-143, C.P. 37150, León, Guanajuato, México*

M. P. Ryan[§]

A spectral action of Euclidean supergravity is proposed. We calculate up to a_4 , the Seeley-Dewitt coefficients in the expansion of the spectral action associated to the supergravity Dirac operator. This is possible because in simple supergravity, as in pure gravity, a well defined and mathematically consistent Dirac operator can be constructed.

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^{*} jllopez@fisica.ugto.mx

[†] octavio@fisica.ugto.mx

[‡] msabido@fisica.ugto.mx

[§] ryanmex2002@yahoo.com Present address: Lane, Millington MD 21651 USA. 30101 Clipper

I. INTRODUCTION

The equivalence principle and gauge invariance are the fundamental pillars of the two most successful theories in physics, general relativity and Yang-Mills theory. They lead to a greater understanding of the basic interactions in our Universe, however these theories seem to be incompatible at the quantum level. This incompatibility might suggest that they are theories arising from some other more fundamental principle. One of the most interesting proposals in the literature is the spectral action of noncommutative geometry. It involves a spectral geometry consistent with the physical measurements of distances. The usual emphasis on the points $x \in \mathcal{M}$ on a geometric space is replaced by the spectrum Σ of the Dirac operator D and it is assumed that the spectral action depends only on Σ . This is the spectral action principle. The spectrum is a geometric invariant that replaces diffeomorphism invariance. By applying this basic principle to the noncommutative geometry defined by the standard model, it has been shown [1] that the dynamics of all interactions, including gravity, are given by the spectral action. Its heat kernel expansion in terms of the Seeley-De Witt coefficients a_n gives an effective action up to the coefficient considered. For the gravitational sector of the spectral action, the first three terms on the expansion correspond to a constant, the usual Einstein-Hilbert action plus Weyl gravity and a Gauss-Bonnet topological invariant.

The Dirac operator is an essential element in the physical action of noncommutative geometry. It encodes, together with representation of the algebra of coordinates, both geometry and physics. It was stated in [2] that in the case of gravity one can consider the eigenvalues of the Dirac operator as observables if they satisfy certain constraints that restrict the phase space and the structure of the space-time manifold. The same type of analysis was later performed for Euclidean supergravity [3, 4] where also the eigenvalues of the appropriate Dirac operator can be understood as observables that must satisfy a set of generalized constraints. We were motivated by this last result and in this work we calculate the first three terms of the spectral action associated with the Dirac operator of simple Euclidean supergravity. First, in section II we review the well known [1] spectral action construction related to pure gravity and then in section III we present the calculation of the first three Seeley-DeWitt coefficients of the heat kernel expansion based on the trace of the square of the supergravity Dirac operator. We write down the entire action up to a_4 and add the Rarita Schwinger term. Finally, section IV is devoted to conclusions and outlook.

II. SPECTRAL ACTION OF PURE GRAVITY

Instead of the well known geometry of space-time, the basic data of noncommutative geometry consists of an involutive algebra \mathcal{A} of operators in a Hilbert space \mathcal{H} , which plays the role of the algebra of coordinates, a self-adjoint operator of Dirac type \mathcal{D} in \mathcal{H} which plays the role of the inverse line element. A fundamental principle in the noncommutative approach is that the usual emphasis on points in space-time is replaced by the spectrum of the operator \mathcal{D} . An operator is of Dirac type if its square is of Laplace type. Locally such operators can be expressed as

$$D = -(g^{\mu\nu} \nabla_\mu \nabla_\nu + E), \quad (1)$$

for a unique endomorphism E acting on vector bundles of \mathcal{M} . The spectral triple $(\mathcal{A}, \mathcal{H}, \mathcal{D})$ encodes the geometry of every noncommutative space. A Riemannian spin manifold \mathcal{M} is completely characterized by the algebra of smooth functions on \mathcal{M} , $\mathcal{A} = C^\infty(\mathcal{M})$, the Hilbert space of square integrable spinors, $\mathcal{H} = L^2(\mathcal{M}, S)$ and the Dirac operator \mathcal{D} of the Levi-Civita spin connection. On Riemannian manifolds \mathcal{D} is an elliptic operator. The selfadjointness and ellipticity of \mathcal{D} is essential for the construction of $(\mathcal{A}, \mathcal{H}, \mathcal{D})$. The spectral action principle states that the physical action depends only on the spectrum of the Dirac operator. These ideas were the origin of the spectral action given in [1]. The bosonic part of the spectral action is

$$S = Tr[f(\frac{D}{\Lambda})]. \quad (2)$$

For a specific choice of the cutoff function f , the spectral action (2) is expressed up to the first three terms of its asymptotic expansion [5]

$$S = Tr[f(\frac{D}{\Lambda})] \sim 2\Lambda^4 f_4 a_0 + 2\Lambda^2 f_2 a_2 + f_0 a_4, \quad (3)$$

where $f_4 = \int_0^\infty f(u) u^3 du$, $f_2 = \int_0^\infty f(u) u du$, $f_0 = f(0)$, and the a_n are the Seeley-Dewitt coefficients of the heat kernel expansion of D . Every a_n is function of geometric invariants of order n constructed from E , the field strength $\Omega_{\mu\nu}$

and the Riemann tensor [6]. The relevant a_n 's are

$$\begin{aligned} a_0 &= \frac{1}{4\pi^2} \int d^4x \sqrt{g}, \\ a_2 &= \frac{1}{16\pi^2} \int d^4x \sqrt{g} \text{Tr}(E + \frac{1}{6}R), \\ a_4 &= \frac{1}{16\pi^2} \frac{1}{360} \int d^4x \sqrt{g} \text{Tr}(12R_{;\mu}^{\mu} + 5R^2 - 2R_{\mu\nu}R^{\mu\nu} \\ &\quad + 2R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + 60RE + 180E^2 + 60E_{;\mu}^{\mu} + 30\Omega_{\mu\nu}\Omega^{\mu\nu}). \end{aligned} \quad (4)$$

For the gravitational Dirac operator \mathcal{D} we have

$$\begin{aligned} \mathcal{D} &= e_a^{\mu} \gamma^a (\partial_{\mu} + \tilde{\omega}_{\mu}) , \\ E &= -\frac{1}{4}R , \\ \Omega_{\mu\nu} &= \frac{1}{4}R_{\mu\nu}{}^{ab} \gamma_{ab} , \end{aligned} \quad (5)$$

where $\tilde{\omega}_{\mu}$ is the spin connection on \mathcal{M} , $\tilde{\omega}_{\mu} = \frac{1}{4}\tilde{\omega}_{\mu}{}^{ab}\gamma_{ab}$ with $\tilde{\omega}_{\mu}$ related to e_{μ}^a by the vanishing of the covariant derivative $\nabla_{\mu}e_{\nu}^a = 0$, this allows us to express the $\tilde{\omega}_{\mu}{}^{ab}$ as functions of the tetrads as in standard Einstein tetradic gravity, applying the Riemannian torsion free condition. The coefficients (4) take the form

$$\begin{aligned} a_0 &= \frac{1}{4\pi^2} \int d^4x \sqrt{g} , \\ a_2 &= -\frac{1}{48\pi^2} \int d^4x \sqrt{g} R , \\ a_4 &= \frac{1}{4\pi^2} \frac{1}{360} \int d^4x \sqrt{g} (-18C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + 11R^*R^*) , \end{aligned} \quad (6)$$

where $C_{\mu\nu\rho\sigma}$ is the Weyl tensor of conformal gravity $C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 2R_{\mu\nu}R^{\mu\nu} + \frac{1}{3}R^2$ and the Euler characteristic χ_E is given by, $\chi_E = \frac{1}{32\pi} \int d^4x \sqrt{g} R^*R^*$ with $R^*R^* = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$, the Gauss-Bonnet topological invariant.

For this particular Dirac operator the spectral action is

$$S = \int d^4x \sqrt{g} \{ \alpha + \beta R + \gamma (-18C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + 11R^*R^*) \} , \quad (7)$$

where α , β and γ are constants. The spectral action gives the Hilbert-Einstein action with corrections. The action above is of particular interest, because this same expression has been considered as a good candidate for a renormalizable and ghost free theory of gravity [7, 8]. In the next section we consider the Dirac operator of $\mathcal{N} = 1$ Euclidean supergravity and calculate the Seeley-Dewitt coefficients associated with its spectral action. The calculation of this action gives the natural supersymmetric extension of the spectral action for gravity and, as mentioned in the introduction, it has already been shown that under certain conditions [3, 4] the eigenvalues of the supergravity Dirac operator can be considered as observables.

III. SPECTRAL ACTION OF EUCLIDEAN SUPERGRAVITY

Let \mathcal{M} be a compact Riemannian spin manifold without boundary in four dimensions with metric $g_{\mu\nu} = e_{\mu}^a e_{\nu a}$, the tetrad fields are labeled with greek space-time and latin internal indices respectively. The Dirac operator \mathcal{D} given by the spin connection in (5), is an elliptic operator on \mathcal{M} and is formally selfadjoint on \mathcal{H} . Because \mathcal{M} is compact, \mathcal{D} admits a discrete spectrum of real eigenvalues and a complete set of eigenspinors $\mathcal{D}\psi^n = \lambda^n \psi^n$. The λ^n 's define a discrete family of real valued functions on the phase space of smooth tetrad fields and as was shown in [2], these eigenvalues are invariant under diffeomorphisms of \mathcal{M} and under rotations of the tetrad fields, so they form a set of observables for general relativity. These ideas were extended in [3, 4] to achieve the geometric construction of Euclidean supergravity. This involving a supersymmetric partner of the graviton, the gravitino, and also imposing local supersymmetric invariance. The gravitino is represented by a Euclidean spinor vector ψ_{μ}^a defined by a Majorana

condition, $\bar{\psi} = \psi^T C$. The phase space will be the space of all pairs (e, ψ) that are solution of the equations of motion modulo gauge transformations, which are the ones needed in the non supersymmetric case plus the transformations of local $\mathcal{N}=1$ supersymmetry. The supersymmetric Dirac operator D_{SG} is given by

$$D_{SG} = i\gamma^a e_a^\mu [\partial_\mu + (\tilde{\omega}_{\mu bc} + K_{\mu bc})\sigma^{bc}]. \quad (8)$$

The only difference between D_{SG} and \mathcal{D} is the additional ψ dependent term [9, 10], $K_{\mu ab} = -\frac{1}{4}(\bar{\psi}_\mu \gamma_b \psi_a - \bar{\psi}_a \gamma_\mu \psi_b + \bar{\psi}_b \gamma_a \psi_\mu)$. D_{SG} is an elliptic operator defined on the full spin bundle and it is possible to define an inner product such that D_{SG} is formally selfadjoint [3]. Here we calculate the Seeley DeWitt coefficients of the square of D_{SG} . The square of this Dirac operator can be expressed in the form of (1) with

$$\begin{aligned} E = & -\frac{1}{4}R - \frac{1}{4}\nabla_\mu(\bar{\psi}^\mu \gamma_\nu \psi^\nu) + \frac{1}{16}\bar{\psi}_\alpha \gamma^\alpha \psi_\beta \bar{\psi}^\nu \gamma_\nu \psi^\beta \\ & + \frac{1}{32}\bar{\psi}^\nu \gamma^\alpha \psi_\beta \bar{\psi}_\alpha \gamma^\beta \psi_\nu - \frac{1}{64}\bar{\psi}_\nu \gamma_\alpha \psi_\beta \bar{\psi}^\nu \gamma^\alpha \psi^\beta, \end{aligned} \quad (9)$$

where R is the curvature scalar of standard general relativity. The field strength is

$$\begin{aligned} \Omega_{\mu\nu} = & \frac{1}{4}R_{\mu\nu}{}^{ab}\gamma_{ab} \\ & + \frac{1}{16}[\bar{\psi}_\mu \gamma^\sigma \psi^a \bar{\psi}_\nu \gamma^b \psi_\sigma - \bar{\psi}_\mu \gamma^\sigma \psi^a \bar{\psi}_\sigma \gamma_\nu \psi^b + \bar{\psi}_\mu \gamma^\sigma \psi^a \bar{\psi}^b \gamma_\sigma \psi_\nu \\ & - \bar{\psi}^a \gamma_\mu \psi^\sigma \bar{\psi}_\nu \gamma^b \psi_\sigma + \bar{\psi}^a \gamma_\mu \psi^\sigma \bar{\psi}_\sigma \gamma_\nu \psi^b - \bar{\psi}^a \gamma_\mu \psi^\sigma \bar{\psi}^b \gamma_\sigma \psi_\nu \\ & + \bar{\psi}^\sigma \gamma^a \psi_\mu \bar{\psi}_\nu \gamma^b \psi_\sigma - \bar{\psi}^\sigma \gamma^a \psi_\mu \bar{\psi}_\sigma \gamma_\nu \psi^b + \bar{\psi}^\sigma \gamma^a \psi_\mu \bar{\psi}^b \gamma_\sigma \psi_\nu] \gamma_{ab} \\ & - \frac{1}{4}\nabla_\mu(\bar{\psi}_\nu \gamma^b \psi^a - \bar{\psi}^a \gamma_\nu \psi^b + \bar{\psi}^b \gamma^a \psi_\nu) \gamma_{ab} - (\mu \leftrightarrow \nu). \end{aligned} \quad (10)$$

Using the calculation of the spectral action for pure supergravity we get the spectral action related to this particular Dirac operator and the constrained geometry defined by it. The first non constant term is

$$\begin{aligned} a_2 = & -\frac{1}{48\pi^2} \int d^4x e(R - \frac{1}{4}\bar{\psi}_\alpha \gamma^\alpha \psi_\beta \bar{\psi}^\nu \gamma_\nu \psi^\beta \\ & - \frac{1}{8}\bar{\psi}^\nu \gamma^\alpha \psi_\beta \bar{\psi}_\alpha \gamma^\beta \psi_\nu + \frac{1}{16}\bar{\psi}_\nu \gamma_\alpha \psi_\beta \bar{\psi}^\nu \gamma^\alpha \psi^\beta). \end{aligned} \quad (11)$$

The term a_4 is a combination of terms quadratic in ψ and non trivial interactions between the graviton and the gravitino

$$\begin{aligned} a_4 = & \frac{1}{4\pi^2} \frac{1}{360} \int d^4x \sqrt{g} [-18C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + 11R^* R^* \\ & - 14R_{\mu\nu\rho\sigma} \Phi^{\mu\nu\rho\sigma}(\psi) - 106R_{\mu\nu} \Sigma^{\mu\nu}(\psi) + 10R\Gamma(\psi) \\ & - 7\Phi_{\mu\nu\rho\sigma}(\psi) \Phi^{\mu\nu\rho\sigma}(\psi) - 62\Sigma_{\mu\nu}(\psi) \Sigma^{\mu\nu}(\psi) + 5\Gamma^2(\psi)]. \end{aligned} \quad (12)$$

The quadratic terms appearing in (12) are given by

$$\begin{aligned} \Phi_{\mu\nu\rho\sigma}(\psi) = & \frac{1}{4}\nabla_\mu(\bar{\psi}_\rho \gamma_\sigma \psi_\nu + \bar{\psi}_\rho \gamma_\nu \psi_\sigma + \bar{\psi}_\nu \gamma_\rho \psi_\sigma) \\ & + \frac{1}{16}(\bar{\psi}_\rho \gamma_\alpha \psi_\mu \bar{\psi}^\alpha \gamma_\sigma \psi_\nu + \bar{\psi}_\rho \gamma_\alpha \psi_\mu \bar{\psi}^\alpha \gamma_\nu \psi_\sigma \\ & + \bar{\psi}_\rho \gamma_\alpha \psi_\mu \bar{\psi}_\nu \gamma^\alpha \psi_\sigma + \bar{\psi}_\rho \gamma_\mu \psi_\alpha \bar{\psi}^\alpha \gamma_\sigma \psi_\nu \\ & + \bar{\psi}_\rho \gamma_\mu \psi_\alpha \bar{\psi}^\alpha \gamma_\nu \psi_\sigma + \bar{\psi}_\rho \gamma_\mu \psi_\alpha \bar{\psi}_\nu \gamma^\alpha \psi_\sigma \\ & + \bar{\psi}_\mu \gamma_\rho \psi_\alpha \bar{\psi}^\alpha \gamma_\sigma \psi_\nu + \bar{\psi}_\mu \gamma_\rho \psi_\alpha \bar{\psi}^\alpha \gamma_\nu \psi_\sigma \\ & + \bar{\psi}_\mu \gamma_\rho \psi_\alpha \bar{\psi}_\nu \gamma^\alpha \psi_\sigma) - (\mu \leftrightarrow \nu), \end{aligned} \quad (13)$$

$$\begin{aligned}
\Sigma_{\mu\nu}(\psi) &= \frac{1}{2}\nabla_\mu(\bar{\psi}_\nu\gamma^\alpha\psi_\alpha) \\
&- \frac{1}{4}\nabla_\alpha(\bar{\psi}_\nu\gamma^\alpha\psi_\mu + \bar{\psi}_\nu\gamma_\mu\psi^\alpha + \bar{\psi}_\mu\gamma_\nu\psi^\alpha) \\
&+ \frac{1}{8}(\bar{\psi}_\nu\gamma_\beta\psi_\mu\bar{\psi}^\beta\gamma^\alpha\psi_\alpha + \bar{\psi}_\nu\gamma_\mu\psi_\beta\bar{\psi}^\beta\gamma^\alpha\psi_\alpha \\
&\quad + \bar{\psi}_\mu\gamma_\nu\psi_\beta\bar{\psi}^\beta\gamma^\alpha\psi_\alpha) \\
&- \frac{1}{16}(\bar{\psi}_\nu\gamma_\beta\psi_\alpha\bar{\psi}^\beta\gamma^\alpha\psi_\mu + \bar{\psi}_\nu\gamma_\beta\psi_\alpha\bar{\psi}^\beta\gamma_\mu\psi^\alpha \\
&\quad + \bar{\psi}_\nu\gamma_\beta\psi_\alpha\bar{\psi}^\beta\gamma^\beta\psi^\alpha + \bar{\psi}_\nu\gamma_\alpha\psi_\beta\bar{\psi}^\beta\gamma^\alpha\psi_\mu \\
&\quad + \bar{\psi}_\nu\gamma_\alpha\psi_\beta\bar{\psi}^\beta\gamma_\mu\psi^\alpha + \bar{\psi}_\nu\gamma_\alpha\psi_\beta\bar{\psi}^\beta\gamma^\beta\psi^\alpha \\
&\quad + \bar{\psi}_\alpha\gamma_\nu\psi_\beta\bar{\psi}^\beta\gamma^\alpha\psi_\mu + \bar{\psi}_\alpha\gamma_\nu\psi_\beta\bar{\psi}^\beta\gamma_\mu\psi^\alpha \\
&\quad + \bar{\psi}_\alpha\gamma_\nu\psi_\beta\bar{\psi}^\beta\gamma^\beta\psi^\alpha),
\end{aligned} \tag{14}$$

$$\begin{aligned}
\Gamma(\psi) &= \nabla_\mu(\bar{\psi}^\mu\gamma^\nu\psi_\nu) - \frac{1}{4}\bar{\psi}_\alpha\gamma^\alpha\psi_\beta\bar{\psi}^\nu\gamma_\nu\psi^\beta \\
&- \frac{1}{8}\bar{\psi}^\nu\gamma^\alpha\psi_\beta\bar{\psi}_\alpha\gamma^\beta\psi_\nu + \frac{1}{16}\bar{\psi}_\nu\gamma_\alpha\psi_\beta\bar{\psi}^\nu\gamma^\alpha\psi^\beta.
\end{aligned} \tag{15}$$

We recognize in the a_2 term the “bosonic” part of the $\mathcal{N}=1$ supergravity action written as a second-order formalism. Now we can write it in terms of the curvature scalar that is function of the spin connection including the torsion term

$$\begin{aligned}
R(e, \psi) &= R - \frac{1}{4}\bar{\psi}_\alpha\gamma^\alpha\psi_\beta\bar{\psi}^\nu\gamma_\nu\psi^\beta - \frac{1}{8}\bar{\psi}^\nu\gamma^\alpha\psi_\beta\bar{\psi}_\alpha\gamma^\beta\psi_\nu \\
&\quad + \frac{1}{16}\bar{\psi}_\nu\gamma_\alpha\psi_\beta\bar{\psi}^\nu\gamma^\alpha\psi^\beta.
\end{aligned} \tag{16}$$

Having modified the Dirac operator in a consistent way, summing the torsion term, the spectral action includes the geometric part of the $\mathcal{N}=1$ supergravity action. The full action is recovered by adding the spin-3/2 action, that is, the Rarita-Schwinger action $(\psi, D_{SG}\psi)$. The whole action (up to a_4) is then a higher order theory of supergravity represented by the spectral action

$$\begin{aligned}
S &= Tr[f(\frac{D_{SG}}{\Lambda})] + (\psi, D_{SG}\psi) \\
&= \int d^4x e[\alpha + \beta R(e, \psi)] + (\psi, D_{SG}\psi) \\
&+ \gamma \int d^4x e[-18C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + 11R^*R^* \\
&- 14R_{\mu\nu\rho\sigma}\Phi^{\mu\nu\rho\sigma}(\psi) - 106R_{\mu\nu}\Sigma^{\mu\nu}(\psi) + 10R\Gamma(\psi) \\
&- 7\Phi_{\mu\nu\rho\sigma}(\psi)\Phi^{\mu\nu\rho\sigma}(\psi) - 62\Sigma_{\mu\nu}(\psi)\Sigma^{\mu\nu}(\psi) + 5\Gamma^2(\psi)],
\end{aligned} \tag{17}$$

with α , β , and γ constants. It is of interest to notice that the spectral action (2) for $D = \mathcal{D}^2$ is a theory of gravitation. It includes in a natural way the Einstein-Hilbert action. However, a general higher order theory of gravity suffers from ghosts, a scalar and a spin-2 mode, both massive. However, the particular theory of gravitation that emerges in the non commutative geometry framework, up to the a_4 term, is of the type

$$I = \frac{1}{2\kappa^2} \int d^4x \left(R - 2\Lambda + \frac{1}{2}\alpha C_{\mu\nu\rho\alpha}C^{\mu\nu\rho\alpha} \right), \tag{18}$$

this form of the theory, without a cosmological constant, was considered first in [11] and it was argued that it is renormalizable. It was then reconsidered in [7] including Λ because in this case the scalar mode is absent and for a special value of α in terms of Λ , the massive spin-2 mode also disappears, leaving a theory consisting only of a massless graviton and is possibly renormalizable. The spectral action we propose (17), is the supergravity spectral action that will correspond to the gravity action (18) in [8]. The action (17) gives to first order usual simple supergravity. It can be written in the form of an Einstein-Weyl supergravity without the scalar and vector auxiliary fields. The Seeley-Dewitt

coefficients have been calculated in particular when totally antisymmetric torsion is present and by these means the associated spectral action [12–17]. The motivations of doing so are based on some physical arguments, for instance the coincidence of geodesics in both manifolds, one in which there is torsion and another where it is absent. However, the supergravity torsion is special and there is no reason, either physical or mathematical, for not considering it because it provides the spectral action associated with simple supergravity.

IV. DISCUSSION

The spectral action allows us to construct a modified theory of gravity as well as a modified theory of supergravity at the desired order in the Seeley-DeWitt coefficients. The fact that makes this possible is that simple supergravity and pure gravity can both be viewed as theories with a well defined, mathematically consistent Dirac operator. This is not, in general, the case for extended supergravities. We were partially motivated by the result that the eigenvalues of the supergravity Dirac operator can, with certain constraints, be considered as observables [3, 4]. We were able to calculate the expansion up to a_4 of the spectral action (2) for the supersymmetric Dirac operator (8), and we have constructed an $\mathcal{N} = 1$ supergravity action (17) by means of this operator. The gravity action (7,18) has been constructed with the pure gravity spin connection (5). This gravity action was already known [8, 11] and may be considered to possibly be renormalizable, whether this could also be the case for the generalized supergravity action (17) is a subject of further study.

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